Econ 802

Final Exam

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Read each question carefully and try to use all of the information provided. All questions have equal weight. If something is unclear, please ask.

1. Consider a competitive firm with one output and n inputs. Assume all functions are differentiable, all solutions are interior, and all questions refer to the long run. Prove mathematically that the following statements are true.

(a) The firm's profit function is convex as a function of the output price when the input price vector is held constant.

(b) If the input vector x* maximizes profit and the resulting profit is positive, then the firm has locally decreasing returns to scale at x*.

(c) The Lagrange multiplier in the firm's cost minimization problem is marginal cost.

2. Consider a competitive firm with one output and 2 inputs. Assume the functions in parts (a) and (b) are differentiable, all solutions are interior, and all questions refer to the long run. Prove mathematically that the following statements are true.

(a) If the production function is CES then the elasticity of substitution is a constant.

(b) If the production function is homothetic then the input expansion path is linear.

(c) If the production function is Leontief and the firm's profit maximization problem has a solution, then the firm's profit must be zero.

3. George has a bliss point at x*. His utility is $u(x_1, x_2) = -(x_1 - x_1^*)^2 - (x_2 - x_2^*)^2$ so he is <u>worse off</u> when his consumption bundle is further away from $(x_1^*, x_2^*) > 0$.

(a) Do these preferences satisfy local non-satiation? Weak monotonicity? Strong monotonicity? Strict convexity? Explain your answers using graphs.

(b) Consider the duality relationship x(p, m) = h[p, v(p, m)] between the Marshallian and Hicksian demands, and the duality relationship e[p, v(p, m)] = m between the expenditure function and the indirect utility function. Do these equalities always, sometimes, or never hold for George? Explain your answers using graphs.

(c) For what set of consumption bundles x does George have a well-defined inverse demand function p(x)? Explain using a graph. For bundles such that p(x) is well-defined, assume income is m = 1 and explain how you would solve for the prices p(x) such that George demands a given bundle x.

4. Each consumer $i = 1 \,..\, n$ has utility $u_i = b_i(x_i) + y_i$ and an endowment m_i of the y good. There are no endowments of the x good. Each firm $j = 1 \,..\, m$ has a cost function $c_j(q_j)$ where q_j is the output of the x good and $c_j(q_j)$ is the input of the y good. Let $b_i(0) = 0$, $b_i' > 0$, $b_i'' < 0$ for all i and $c_j(0) = 0$, $c'_j > 0$, $c_j'' > 0$ for all j.

(a) A social planner wants to maximize the sum of the utilities. Derive the first order conditions and use a graph to show how the planner could solve for the Lagrange multiplier λ for the feasibility constraint on the x good. Interpret your graph.

(b) Let p be the price of the x good and set the price of the y good equal to one. Each consumer i gets a share $T_{ij} \geq 0$ in the profit of firm j, where $\sum_i T_{ij} = 1$ for all j. Let $z_x(p)$ and $z_y(p)$ be aggregate excess demand functions for the x and y good. Show that $pz_x(p) + z_y(p) = 0$ for all values of p, and interpret this result.

(c) For n = 2 (two consumers but any number of firms), show the Pareto frontier on a graph with u_1 on the horizontal axis and u_2 on the vertical axis. Pick an arbitrary point on the frontier and show that it can be reached using Walrasian equilibrium.

5. There are n consumers $i = 1 \,..\, n$. Consumer i's utility is $u_i = a_i \ln r_i + b_i \ln L_i$ where r_i is i's consumption of rhubarb, L_i is leisure time, and $a_i > 0$, $b_i > 0$. Each consumer is endowed with one unit of time that can be used either for leisure or labor. There are no rhubarb endowments. There are firms $j = 1 \,..\, m$. Each firm has the same production function $y_j = kz_j$ where y_j is the firm's output of rhubarb, z_j is the firm's input of labor, and $k > 0$ is a constant. Each consumer i receives an equal share 1/n in the profit π_j of firm j, for all $j = 1 \,..\, m$. The price of rhubarb is p and the price of labor is w.

(a) Write out agent i's budget constraint. Define the agent's income m_i and solve for the agent's Marshallian demands for rhubarb and leisure as functions of (p, w).

(b) Find a Walrasian equilibrium price vector (p, w), the total output of rhubarb, the total demand for labor, and the total demand for leisure. Justify your answers.

(c) Let n = m = 1 so there is one consumer and one firm. Continue to assume price-taking behavior. Draw a graph with leisure on the horizontal axis and rhubarb on the vertical axis. Show the Walrasian equilibrium allocation on this graph and discuss the nature of the budget constraint and the isoprofit lines. Also explain the nature of the firm's profit maximization problem and how the markets clear.